

Nonsingular static global string

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Abstract

A new solution for the spacetime outside the core of a $U(1)$ static global string has been presented which is nonsingular. This is the first example of a nonsingular spacetime around a static global string.

It is believed that early universe has undergone a number of phase transitions as it cooled down from the hot initial phase. One of the immediate consequences of these phase transitions is the formation of defects or mismatches in the orientation of the Higgs field in causally disconnected regions [1]. One of these defects, cosmic string, is particularly interesting as it is capable of producing observational effects and also may play an important role in the large scale structure formation of the universe. The

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gravitational field of a static infinitely long gauge string, produced by the breaking of a $U(1)$ local symmetry, has been studied in great detail [2]. It has been shown in all of these investigations that the spacetime around an infinitely long local gauge string is Minkowskian minus a wedge. However for global strings which arise due to the breaking of global symmetry, the exact gravitational field is not that simple. Global strings are such that their energy extends to regions far beyond the central core, the energy density being proportional to r^{-2} , which leads to a logarithmically divergent mass per unit length. For a global string, Harari and Sikivie [3] presented a solution of the linearised Einstein's field equations neglecting the radial variation of the scalar field outside the core of the string. That the approach was not self-consistent became evident as there is a physical singularity at a finite distance from the string core. Cohen and Kaplan [4] considered the exact nonlinear field equations. In their solution, there was an unexplained singularity on the symmetry axis while the string appeared as a singular cylindrical surface of finite radius. The present authors have considered [5] the Einstein's field equations outside the core of a static global string, in a coordinate system different from that considered by Cohen and Kaplan. It has been shown that our solution reduces to the Cohen and Kaplan solution under a coordinate transformation. That an undesirable singularity is unavoidable for the static global string was later pointed out by Gregory [6] who ascribed it to a peculiarity of the energy momentum tensor rather than to the slow fall off of the energy density with radial distance. Gibbons et.al showed that the global string cannot admit any asymptotically well behaved solution as the deficit angle produced by the string diverges logarithmically for a large distance [7]. Later on, Gregory [8] showed that one can get a nonsingular solution for the spacetime outside the core of a global string only if one includes a time dependence along the symmetry axis. It should be mentioned that in all of these works, it has been assumed that the spacetime admits a Lorentz boost along the symmetry axis. In the present work, we have been able to find a nonsingular solution for the spacetime outside the core of a global string. For achieving such a solution, we have relaxed the condition of a Lorentz boost being admitted along the symmetry axis of the string.

To describe the spacetime geometry due to an infinitely long static cosmic string,

the line element is taken to be the general static cylindrically symmetric one given by

$$ds^2 = e^{2(K-U)}(dt^2 - dr^2) - e^{2U}dz^2 - e^{-2U}W^2d\theta^2, \quad (1)$$

where K, U, W are functions of r alone. For a global string, the energy momentum tensor components are calculated from the action density for a complex scalar field ψ along with a Mexican hat potential:

$$L = \frac{1}{2}g^{\mu\nu}\psi_{,\mu}^*\psi_{,\nu} - \frac{\lambda}{4}(\psi^*\psi - v^2)^2, \quad (2)$$

where λ and v are constant and $\delta = (v\sqrt{\lambda})^{-1}$ is a measure of the core radius of the string. It has been shown that [6] the field configuration can be choosen as

$$\psi(r) = vf(r)e^{i\theta} \quad (3)$$

in cylindrical field coordinates. The usual boundary condition on $f(r)$ is $f(0) = 0$ and $f(r) \rightarrow 1$ as $r \rightarrow \delta$. As we are interested in spacetime outside the core of the string, for our purpose

$$f(r) = 1, f'(r) = 0 \quad (4)$$

is a good approximation. The nonzero components of the energy momentum tensor outside the core of the string now become

$$T_t^t = T_r^r = T_z^z = -T_\theta^\theta = \frac{v^2}{2} \frac{e^{2U}}{W^2}. \quad (5)$$

The Einstein's equations, $G_\nu^\mu = 8\pi T_\nu^\mu$, are

$$-\frac{W''}{W} + \frac{K'W'}{W} - U'^2 = -\frac{4\pi v^2}{W^2}e^{2K}, \quad (6a)$$

$$-\frac{K'W'}{W} + U'^2 = -\frac{4\pi v^2}{W^2}e^{2K}, \quad (6b)$$

$$-K'' - U'^2 = \frac{4\pi v^2}{W^2}e^{2K}, \quad (6c)$$

$$-\frac{W''}{W} + 2U'' + 2U'\frac{W'}{W} - K'' - U'^2 = -\frac{4\pi v^2}{W^2}e^{2K}. \quad (6d)$$

By adding equations (6a) and (6b) one can get

$$\frac{W''}{W} = \frac{8\pi v^2}{W^2}e^{2K}. \quad (7a)$$

Again using (6c) and (6d) one can write

$$-\frac{W''}{W} + 2U'' + 2U'\frac{W'}{W} = -\frac{8\pi v^2}{W^2}e^{2K}. \quad (7b)$$

Now adding (7a) and (7b) one gets

$$U'' + \frac{U'W'}{W} = 0,$$

which on integration yields

$$U' = \frac{\alpha}{W}, \quad (8)$$

where α is an integration constant. Again by adding (6b) and (6c) one can get

$$K'' + \frac{K'W'}{W} = 0,$$

which on integration yields

$$K' = \frac{\beta}{W}, \quad (9)$$

where β is another integration constant. Now putting (8) and (9) in (6c) one can write

$$K'' + (A^2 + B^2e^{2K})K'^2 = 0, \quad (10)$$

where $A^2 = \frac{\alpha^2}{\beta^2}$ and $B^2 = \frac{4\pi v^2}{\beta^2}$. For any arbitrary value of A and B it is very difficult to obtain a solution for (10) in closed form. However if one defines a new coordinate

$$\frac{u}{u_0} = \exp(k),$$

then one can find a solution for the complete spacetime which becomes

$$ds^2 = (u/u_0)^{2(1-A)}dt^2 - (u/u_0)^{2A}dz^2 - (u/u_0)^{2A(A-1)}\exp[B^2(u/u_0)^2][(1/u_0^2)du^2 + \beta^2d\theta^2] \quad (11a)$$

This solution is similar to that obtained earlier by Cohen and kaplan [4] for global string with bound matter at rest. The spacetime may have singularity at $u = 0$ or at $u = \infty$ depending on the value of A which is an arbitrary constant.

For $A = 1/2$ one can check that the spacetime admit the Lorentz boost and space-time (11a) becomes

$$ds^2 = (u/u_0)(dt^2 - dz^2) - (u/u_0)^{-1/2}\exp[B^2(u/u_0)^2][(1/u_0^2)du^2 + \beta^2d\theta^2] \quad (11b)$$

This is similar to that obtained by Cohen and Kaplan [4] and also by present authors [5] and also has a singularity at finite distance outside the core. But one may notice that there is one difference between the metric (11a) and (11b) with the corresponding solutions given by Cohen and Kaplan (CK). In CK solution, the exponential term in g_{uu} and in $g_{\theta\theta}$ appears as the reciprocal of what we get here.

However, for $A^2 = 2$, one can get a closed form solution for the equation (10) which is of the form

$$e^{2K} = P + \frac{2}{B^2} \ln(r/r_0), \quad (12)$$

where $r_0 = 2/B = \beta/(\sqrt{\lambda}v)$, and P is an arbitrary constant of integration. The other metric components in this case are

$$e^{2U} = [P + \frac{2}{B^2} \ln(r/r_0)]^{\sqrt{2}}, \quad (13)$$

$$W = 2B\beta(r/r_0)[P + \frac{2}{B^2} \ln(r/r_0)]. \quad (14)$$

The line element can now be written as

$$ds^2 = [P + \frac{2}{B^2} \ln(r/r_0)]^{(1-\sqrt{2})} (dt^2 - dr^2) - [P + \frac{2}{B^2} \ln(r/r_0)]^{\sqrt{2}} dz^2 \\ - 4B^2\beta^2(r/r_0)^2 [P + \frac{2}{B^2} \ln(r/r_0)]^{(2-\sqrt{2})} d\theta^2. \quad (15)$$

One can identify r_0 as the core radius δ of the string and hence the above line element is valid for $r \geq r_0$. Now if P , which is an arbitrary integration constant, is positive, then it can be shown that there is no singularity for $r \geq \delta$. To check this, one can calculate the Kretschman scalar for the metric (1) which comes out as

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} = [\{(1-\sqrt{2})K'' + 2(1-\sqrt{2})^2K'^2\}^2 + (\sqrt{2}K'' + 4K'^2)^2 + \{\frac{W''}{W} - \sqrt{2}K'' - \\ 4\sqrt{2}\frac{K'W'}{W} + \frac{W'^2}{W^2} + 4K'^2\}^2]e^{4(U-K)}. \quad (16)$$

Now the different terms in the expression (16) look like

$$K' = \frac{1}{B^2r[P + \frac{2}{B^2} \ln(r/r_0)]}, \quad (17a)$$

$$K'' = -\frac{P + \frac{2}{B^2} + \frac{2}{B^2} \ln(r/r_0)}{B^2r^2[P + \frac{2}{B^2} \ln(r/r_0)]^2}, \quad (17b)$$

$$\frac{W'}{W} = \frac{P + \frac{2}{B^2} + \frac{2}{B^2} \ln(r/r_0)}{r[P + \frac{2}{B^2} \ln(r/r_0)]}, \quad (17c)$$

$$\frac{W''}{W} = \frac{2}{B^2 r^2 [P + \frac{2}{B^2} \ln(r/r_0)]}. \quad (17d)$$

From expressions (17a) - (17d) and also from the expressions for e^{2U} and e^{2K} one can check easily that the Kretschmann scalar (16) approaches zero as $r \rightarrow \infty$ and for all other value of $r > r_0$ the scalar is finite. Hence one can conclude that the metric (15) for the spacetime outside the core of a global string is nonsingular. This is perhaps the first example of a nonsingular spacetime outside the core of a static global string. For obtaining such a solution we have relaxed the condition of admitting a Lorentz boost. Gregory has shown [6] that spacetime of global string should admit Lorentz boost if one has to demand elementary flatness on the axis of the string. But as our spacetime is valid only for outside the string core one can not demand elementary flatness on the string axis. Hence Lorentz boost is not an essential condition for spacetime outside the string core. Another important thing is that for our spacetime (15) one can have $P_r + P_\theta = 0$ where P_r and P_θ are the pressures along the radial and tangential direction respectively and thus Gregory's conjecture, that this peculiarity of the energy momentum tensor leads to the singular nature of global string spacetime, does not hold. It can also be checked that the energy density T_t^t goes to zero as $r \rightarrow \infty$ and also the kretschmann scalar becomes zero for $r \rightarrow \infty$ as we have mentioned earlier. Hence the solution is asymptotically well behaved which removes the unpleasant feature that a global string can not be asymptotically well behaved [7]. It appears that all these properties are the result of relaxing the demand of a Lorentz boost along the symmetry axis. To preserve the Lorentz boost, One has to take $A^2 = 1/2$. For all other values of A , this symmetry is lost. we can get the solutions in a closed form only for $A^2 = 2$. For all other values of A , a series solution in the form of $r = r(K)$ may be obtained (like the one presented for $A^2 = 1/2$ in ref [5]), which may or may not have the singularity. In this work, we have thus been able to prove that at least one nonsingular static global string solution can be obtained in the absence of Lorentz boost along the z-axis. A more general study in this line will definitely be worthwhile.

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